Project Objective

The limitations of the existing methods in representing and modelling multidimensional data represent the motivation behind this project, which is a natural continuation of the work commenced during the PhD studies of the project leader. The aim is to address the multilinear system identification problem and to search for a general solution, based on a tensor modelling approach. In contrast with the classical multilinearity definition, which refers to an input-output relation (with respect to the data), the term is used here in the context of a multiple-input/single-output (MISO) system, with respect to the impulse responses of a multidimensional model. For this reason, it is more appropriate to call the systems under study multilinear in parameters.

In this research project, we also focus on the Kronecker product decomposition approach, together with low-rank approximation, to improve the efficiency of the adaptive algorithms used for such challenging system identification problems. The basic idea is to exploit the impulse response decomposition in the multilinear case. This represents an extension of the NKP technique, which was only suitable for cases when the impulse response was decomposable into sums of Kronecker products between two shorter impulse responses. In other words, a high-dimension system identification problem will be reformulated in terms of many low-dimension problems (i.e., shorter filters), which are combined together via the Kronecker product. The expected gain would be twofold, i.e., in terms of both performance and complexity.

Objective 1. Developing an iterative Wiener filter for multilinear in parameters systems

The Wiener filter is the straight-forward solution to the optimization problem of minimizing the mean-square-error (MSE) cost function. It yields the optimal coefficients of the estimated system impulse response, under the stationarity assumption. It is therefore the most natural starting point for any development in the field. We will start by defining the system model in the general multilinear case, and in this context, we will develop the conventional Wiener solution. The following step is to take advantage of the tensorial decomposition of the impulse response using the Kronecker product and, in this way, to develop an iterative version of the Wiener filter, which is expected to achieve both a better value of the misadjustment (i.e., better accuracy of the solution) and a faster convergence as compared to the conventional one. The performance of the proposed approach will be validated through Matlab simulations.

Objective 2. Developing LMS-based algorithms for multilinear in parameters systems

The Wiener solution is a very good option for the case when the systems are time invariant. However, in most real-world applications, the impulse responses change their values over time. Moreover, the previously described Wiener solution may be inappropriate for practical use, due to its high computational complexity, yielded by the matrix inversions and covariance matrix estimations. Therefore, the next step is to develop adaptive filters suited for the problem, and in this context, the LMS algorithm is probably the most popular one. Consequently, our second objective is the development of an LMS algorithm for multilinear system identification, based on tensor decompositions. In this framework, a normalized LMS (NLMS) solution will also be developed, by using a time-dependent step-size parameter. A full convergence analysis is also necessary to complete the developments.

Objective 3. Developing APAs for multilinear in parameters systems

The APA may be regarded as a generalization of the NLMS algorithm, which is obtained through the minimization of several values of the a posteriori error, instead of a single one, as it is the case for NLMS. Since it offers improved convergence features, especially for highly correlated inputs, we will extend the ideas behind the LMS-type algorithms for APA. This is not a straightforward step, since the inputs of APA appear in a matrix form, which raises some difficulties when incorporating into the optimization problem mentioned before. The resulting APA for multilinear systems would inherit the optimized features of the LMS-type algorithms and the fast convergence of APA, thus being suitable especially for acoustic applications.

Objective 4. Developing a KF for multilinear in parameters systems

The KF is a popular tool which is used in many fields, among which system identification is of interest for us. It involves the estimation of a set of unknown coefficients based on a set of noisy measurements. In the context of the KF, we consider that the n individual impulse responses which compose the multilinear system follow first order Markov models, with the uncertainties comprised in vectors describing the systems. Based on our previous research on the KF, we expect it to outperform the APA and even the recursive-least-squares (RLS) algorithm. In order to reduce the significant computational complexity of the KF, we will also develop a simplified version of it (SKF), based on the assumption that the individual misalignments become uncorrelated when the algorithm starts to converge. This assumption is expected to simplify the expressions of the matrices involved in the KF update relations. Moreover, a thorough analysis of the model uncertainties will be conducted, with the purpose of further increasing the performance, obtaining in this way an improved version of the KF with individual control factors.

Objective 5. Developing combinations of adaptive filters for multilinear system identification

The combinations of adaptive filters (having different characteristics or parameters) represent an attractive method to improve the overall performance of the algorithms. In this context, we aim to further improve the performance of the developed algorithms, by using different combination schemes, according to the specific of the applications. The combination layer would select and give more weight to the best filter at every time instant, making parameter selection less critical. For example, an interesting combination could involve the LMS and Kalman-based algorithms tailored for multilinear systems (which have different convergence characteristics). Also, other possible combinations could target LMS-type algorithms using different convergence parameters (to compromise between convergence rate and misadjustment), or APAs using different projection orders. In this framework, we will also focus on providing strategies to reduce the cost of combined schemes, trying to develop new structures whose complexity is close to that of an individual filter.